Accelerators for ion-beam therapy

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Introduction

Particle accelerators are an obvious technical requirement for ion-beam therapy. This exercise sheet should help you understand why different accelerator types have established themselves as standard solutions for particular purposes.

These aspects will be also be discussed in the accelerator lecture on Thursday afternoon. However, try to solve at least some of the problems on your own beforehand, using the references given. You are not expected to be trained accelerator physicists, though general knowledge of mechanics and electrodynamics is required. Do not forget that ions used for hadron therapy tend to have fairly high velocities, so relativistic effects are usually not negligible.

While it is always good to understand things analytically, use your laptops for the heavy computation work — that's what they're here for. Some questions are really not supposed to be answered via pen-and-paper only, and trying to do so will remove all the fun, which is the opposite of what we are trying to achieve.

During your presentation on Friday, avoid "recalculating" your solutions line-by-line (you won't have time for that). Rather, try to reflect your findings and communicate them to your fellow students in an enjoyable way.

1 Drift-tube linac

An Alvarez-like linear accelerator is used to accelerate protons from an initial kinetic energy of 200 keV. The accelerator operates at a frequency of 200 MHz and we assume all gaps to have the same effective gap voltage of 200 kV.

- What is the required length of the n-th gap/drift-tube pair?
- Neglecting all problems related to focussing of the beam, what overall length of the machine is required to reach a final kinetic energy of 10 MeV?
- How long would this machine need to be in order to produce 220-MeV protons for hadron therapy? What parameters would you change in order to obtain a more compact machine?

Make yourselves familiar with the working principle of a DTL using the lecture given by D. Alesini at the 2021 CERN Accelerator School [1]. You will need the introductory sections only.

Solution

- From

$$\gamma mc^2 = E_{\mathrm{kin}} + mc^2 \quad \rightarrow \quad v = c\sqrt{1 - \left(\frac{1}{E_{\mathrm{kin}}/(mc^2) + 1}\right)^2}$$

where m is the proton rest mass, γ the Lorentz factor, and E_{kin} the kinetic energy.

After the n-th gap of $U = 200\,\mathrm{kV}$, $E_{\mathrm{kin}} = (n+1)Ue$. Synchronization with the RF requires $v_n/l_n = f_{\mathrm{rf}}$, where v_n is the velocity after the n-th gap, and l_n is the length of the n-th gap/drift-tube section. Hence

$$l_n = \frac{c}{f_{\mathrm{rf}}}\sqrt{1 - \left(\frac{1}{(n+1)Ue/(mc^2)+1}\right)^2} .$$

- A final kinetic energy of 10 MeV is reached after 49 gaps. Neglecting additional space required for focussing elements, the overall length of the 10-MeV accelerator is

$$L_{10} = \sum_{n=0}^{49} l_n \approx 7.4\,\mathrm{m} .$$

- 220 MeV requires 1099 gaps, hence

$$L_{220} = \sum_{n=0}^{1099} l_n \approx 685\,\mathrm{m} .$$

Increasing U reduces the required number of gap/drift-tube pairs to reach a given final energy. Increasing the cavity frequency reduces the distance from one gap to the next. Note that a real linac of this scale would not consist of a single resonant structure but of many independent cavities.

2 Cyclotron

A synchro-cyclotron is used to produce 220-MeV protons for hadron beam therapy. The accelerator operates at a magnetic flux density of 2.2 T which we assume to be homogeneous. The peak Dee voltage is 100 kV vs. ground.

- How many revolutions do the protons perform in the machine? What is the final cyclotron radius just before particle extraction? Make a plot of the radial separation between neighbouring orbits as a function of turn number. How would you *extract* the beam after its final turn?
- Using the same magnetic field strength, you want to scale-up your machine such that it can deliver 430-MeV/u $^{12}C^{6+}$-ions. How do the above quantities change? Try to identify the reasons why — as of today — no cyclotron suitable for carbon beam therapy has been built yet.

For an overview of how a cyclotron works, have a look at the lecture notes provided by M. Seidel in the context of the 2021 introductory CAS [2]. Therein, you will also find a discussion of the step-size-problem.

Solution

- From the cyclotron frequency

$$\omega_{\mathrm{cyc}} = \frac{v}{\rho} = \frac{qB}{\gamma m} \quad \rightarrow \quad \rho = \frac{\gamma m v}{qB} = \frac{p}{qB} ,$$

where $q = Qe$ is the charge of the particle, and m its rest mass.

The kinetic energy after n turns is $E_{\text{kin}} = 2n(2Uq)$, with U being the peak Dee potential. Thus, 550 turns are necessary to accelerate protons to 220 MeV.

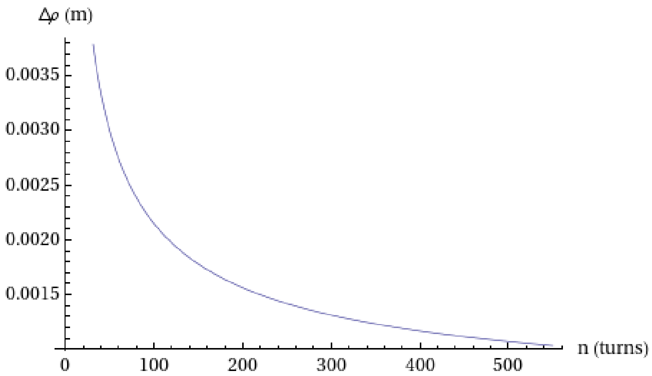
From $pc = \sqrt{E^2 - m^2c^4}$ and using $E = E_{\text{kin}} + mc^2$, we obtain the orbit radius after n turns:

$$\rho = \frac{\sqrt{E_{\text{kin}}^2 + 2E_{\text{kin}}mc^2}}{qBc} = \frac{\sqrt{(4nUq)^2 + 8nUq\,mc^2}}{qBc}$$

For protons after $n = 550$ turns, we get $\rho \approx 1.0\,\text{m}$.

```
In[16]:= Plot[ρ[n] - ρ[n - 1], {n, 0, 550}, AxesLabel → {"n (turns)", "Δρ (m)"}]
```

Out[16]=



The radial separation between neighbouring rounds decreases with rising energy and reaches $\sim 1\,\text{mm}$ at 220 MeV, making it difficult to extract the beam with high efficiency. Even though electrostatic extraction septa can have very thin walls, neighbouring rounds can actually overlap with so small step size. As a remedy, one can, for example, have the field strength decrease towards larger radii, at the expense of an overall larger machine.

- For $m = 12\,\text{u}$ and $q = 6\,e$, a final kinetic energy $E_{\text{kin}} = 12 \times 430\,\text{MeV} = 5.16\,\text{GeV}$ is reached after 2150 turns. The final orbit radius of the carbon ions is then $\rho \approx 3.0\,\text{m}$. The radial separation is below 0.8 mm for the last two rounds.

  The overall size of the magnet bore becomes very large. Modern superconductor technology can reach higher fields, which allows to down-size the machine somewhat. However, the turn separation then becomes even smaller, further complicating beam extraction.

3 Synchrotron

A beam of $^{12}C^{6+}$ ions is injected into a synchrotron at an initial kinetic energy of 7 MeV/u. The synchrotron's bending dipole magnets create fields of $B_{\text{inj}} = 0.16\,\text{T}$ and deflect the beam by 30° each.

- What is the magnetic length of a single dipole magnet? Assuming that the bending magnets cover 46 % of the circumference of the ring, what is the overall length of the closed orbit?
- During acceleration, the bending fields are ramped-up to a final value of $B_{\text{fin}} = 1.39\,\text{T}$, at a constant rate of 1.2 T/s. What is the final kinetic energy of the $^{12}C^{6+}$ ions?
- Give an expression for a synchronous frequency ramp f_{RF} of the accelerating gap as a function of the magnetic field B. Then, make plots of $B(t)$ and $f_{\text{RF}}(t)$ as a function of time.

- Have another look at the linac from Exercise 1. Can you propose a DTL based on the same RF technology that could serve as a 7-MeV/u $^{12}C^{6+}$-injector for our synchrotron? How long would it need to be?

Have a look at the 2015 CAS lecture by B. J. Holzer for an introduction to ion synchrotrons [3]. However, do not dig too deep! You won't actually need any new formalisms with respect to the previous exercises here. Beware that most synchrotrons in the world are electron machines. Hence, random texts and formulae you find online sometimes use ultra-relativistic approximations that are not applicable to ions.

Solution

- The bending radius ρ in the synchrotron dipoles is given by the same equation derived for the cyclotron above. At 7 MeV/u, we have $E_{\text{kin}} \ll mc^2$, hence classical approximations are in order. With $E_{\text{kin}} = 12 \times 7.0\,\text{MeV}$, $q = 6e$, $m = 12\,\text{u}$, and $B = 0.16\,\text{T}$ we get

$$\rho = \frac{p}{qB} = \frac{\sqrt{2E_{\text{kin}}m}}{qB} \approx 4.78\,\text{m}\,,$$

  The length of the arc-of-circle described by an ion in a single dipole is thus

$$l_{\text{dip}} = 2\pi\rho\frac{30^\circ}{360^\circ} = 2.50\,\text{m}\ .$$

  With 30°-bends, 12 dipole magnets are required for closing the orbit. If 54 % of the beam line are straight sections used for other equipment (focussing quads, accelerator cavity, injection and extraction ...), the total circumference of the ring is

$$C = (12 \times l_{\text{dip}}) \times 100/46 = 65\,\text{m}\ .$$

- As we do not know the final energy, we need to switch to relativistic expressions, and find

$$E_{\text{kin}} = \sqrt{(pc)^2 + m^2c^4} - mc^2 = \sqrt{(\rho qBc)^2 + m^2c^4} - mc^2 \approx 5.15\,\text{GeV}\,,$$

  for $B = B_{\text{fin}} = 1.39\,\text{T}$, equivalent to 430 MeV/u.

- From $p = \gamma mv = \rho qB$ we obtain the relation between the particle velocity v and the dipole field B:

$$v = \sqrt{\frac{1}{\left(\frac{m}{\rho qB}\right)^2 + \frac{1}{c^2}}}\ .$$

  In the simplest case (bunching at harmonic number 1), the frequency of the RF gap matches the revolution frequency in the ring:

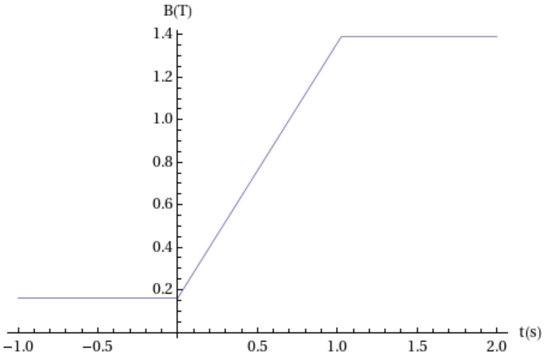
$$f_{\text{rf}} = \frac{v}{C}\ .$$

  As the magnetic field is proportional to the particle momentum, the latter also increases linearly during the acceleration. However, the required gap frequency is proportional to the particle *velocity*. For higher energies, the relativistic mass increase "eats up" some of the momentum increase, so the frequency ramp has to slow down with respect to the magnetic field ramp.

```
In[10]:= Bramp[t_] := 0.16 /; t < 0
         Bramp[t_] := 0.16 + 1.2 * t /; 0 ≤ t ≤ (1.39 - 0.16) / 1.2
         Bramp[t_] := 1.39 /; t > (1.39 - 0.16) / 1.2

In[13]:= Plot[Bramp[t], {t, -1, 2}, AxesLabel → {"t(s)", "B(T)"}]
```

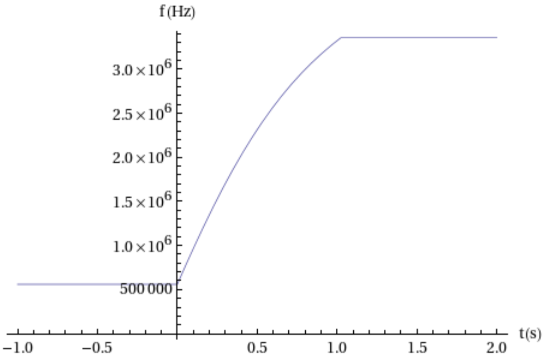
Out[13]=




```
In[14]:= Framp[t_] := Sqrt[1 / ((m / (ρ * q * Bramp[t]))^2 + 1 / c^2)] / 65

In[15]:= Plot[Framp[t], {t, -1, 2}, AxesLabel → {"t(s)", "f(Hz)"}]
```

Out[15]=



- DTLs are nowadays the most common pre-accelerator for ion synchrotrons. At a working frequency of 200 MHz and 200 kV effective gap voltage, the linac would need 69 gaps to accelerate $^{12}C^{6+}$ to the injection energy of 7.0 MeV/u. With the assumptions from Exercise 1, it would be approx. 8.6 m long.

References

[1] D. Alesini, LINAC, *CERN Accelerator School: Introduction to Accelerator Physics*, Chavannes de Bogis, Switzerland, 2021, `https://cas.web.cern.ch/previous-schools`. Direct link: `https://cernbox.cern.ch/s/GJqKGixSYILWfsr`

[2] M. Seidel, Cyclotrons and Fixed Field Alternating Gradient Accelerators, *CERN Accelerator School: Introduction to Accelerator Physics*, Chavannes de Bogis, Switzerland, 2021, `https://cas.web.cern.ch/previous-schools`. Direct link: `https://cernbox.cern.ch/s/GJqKGixSYILWfsr`

[3] B. J. Holzer, Beam Dynamics in Synchrotrons, *CERN Accelerator School: Accelerators for Medical Applications*, Vösendorf, Austria, 2015, `https://cas.web.cern.ch/previous-schools`. Direct link: `https://e-publishing.cern.ch/index.php/CYRSP/issue/view/33`